Bayesian Models for Methodologies of Appraisal

Write $H = C \,\&\, B$,

H: theory; C: hard-core; B: protective belt

Initially all possible H's (C's, B's) are "filtered" w.r.t. background knowledge G

Consider a new prediction X of H

Bayes' theorem gives

$$P(H/X\&G) = \frac{P(X/H\&G)\,P(H/G)}{P(X/H\&G)P(H/G) + P(X/\sim H\&G)P(\sim H/G)}$$

Now omitting G for simplicity of notation

$$= \frac{P(X/H)\,P(H)}{P(X/H)\,P(H) + P(X/\sim H)(1-P(H))}$$

since $P(H) = 1 - P(\sim H)$

$$= \frac{x}{x + \varepsilon(1-x)} = \frac{1}{1-\varepsilon+\varepsilon/x}$$

since $P(X/H) = 1$ and we write $P(H) = x$, $P(X/\sim H) = \varepsilon$

Now suppose all theories to be considered are "filtered" w.r.t. X, i.e. X is not included in G, and we consider a new prediction X' and so on.

Writing P_n for $P(H / X_1 \& X_2 \cdots \& X_n \& a)$

we have by iteration

$$P_n = \text{Lt}_{s\to\infty} \Omega^{(n)} \times \Omega^{(n-1)} \cdots \Omega^{(1)} \cdot P_0$$

where $P_0 = x$

and $$\left.\begin{aligned} \Omega^{(s)} &= \frac{1}{P_{s_1}(1-\varepsilon)+\varepsilon} \\ P_{s+1} &= \Omega^{(s+1)} \cdot P_s \end{aligned}\right\}$$

Soln of this recursion is by inspection (a more scruffy way by replacing $\varepsilon \to \varepsilon^n$ in formula for P_1)

thus $$P_n = \frac{1}{1-\varepsilon^n + \varepsilon^n/x} \qquad \text{——(1)}$$

We can also ask what is prob. for $n+1^{\text{st}}$ prediction X_{n+1} after n successful predictions. Denote this by $P(X_{n+1})$

Clearly $P(X_{n+1}) = P(X_{n+1}/H)\,P_n(H) + P(X_{n+1}/\sim H)\,P_n(\sim H)$

$$= P_n(H) + \varepsilon\,(1 - P_n(H))$$

$$P(X_{n+1}) = \varepsilon + \frac{1-\varepsilon}{1-\varepsilon^n + \varepsilon^n/x} \qquad \text{——(2)}$$

Since ε is small, e.g. take $\varepsilon = .1$, $x = .01$

we can approximate

$$P_n \approx \frac{1}{1 + \varepsilon^n/x}$$

$$P(X_{n+1}) \approx \varepsilon + \frac{1}{1 + \varepsilon^n/x} = \varepsilon + P_n$$

also $\Lambda^{(n)} = P_n / P_{n-1} \approx \dfrac{1}{P_{n-1} + \varepsilon}$

$\Lambda^{(n)}$ is the "enhancement" factor

Note

① If P_{n-1} is $\ll \varepsilon$, then $\Lambda^{(n)} \sim 1/\varepsilon$

as P_{n-1} builds up $\to 1$, $\Lambda^{(n)} \to 1$.

② for $P_n \ll \varepsilon$, $P(X_{n+1}) \sim \varepsilon$

as P_n builds up $\to 1$, $P(X_{n+1}) \to 1$

③ for $\varepsilon^n/x \gg 1$ P_n is small

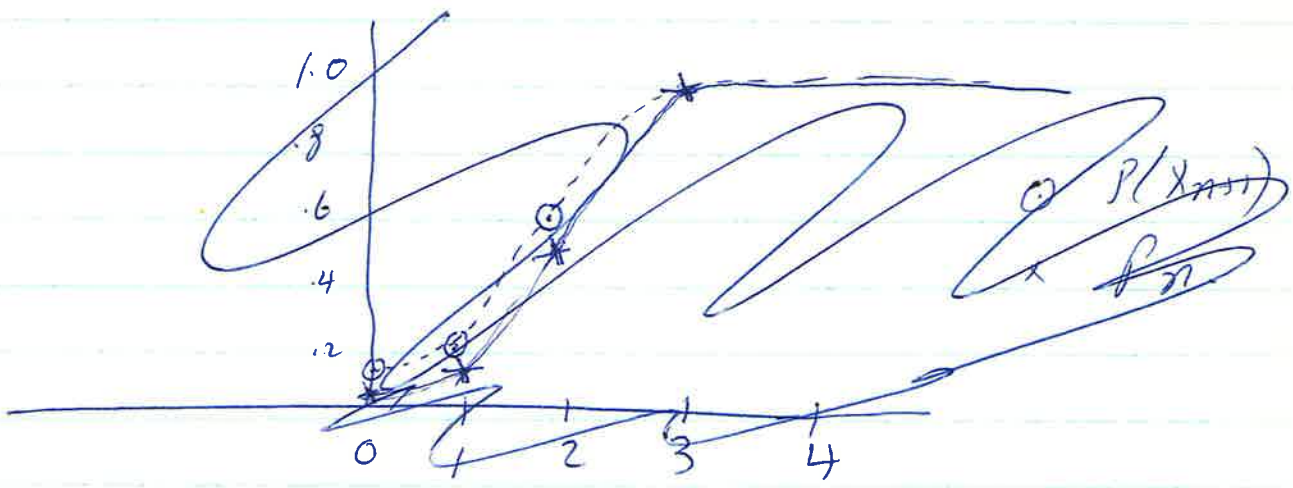
$\varepsilon^n/x \ll 1$ P_n is ~ 1

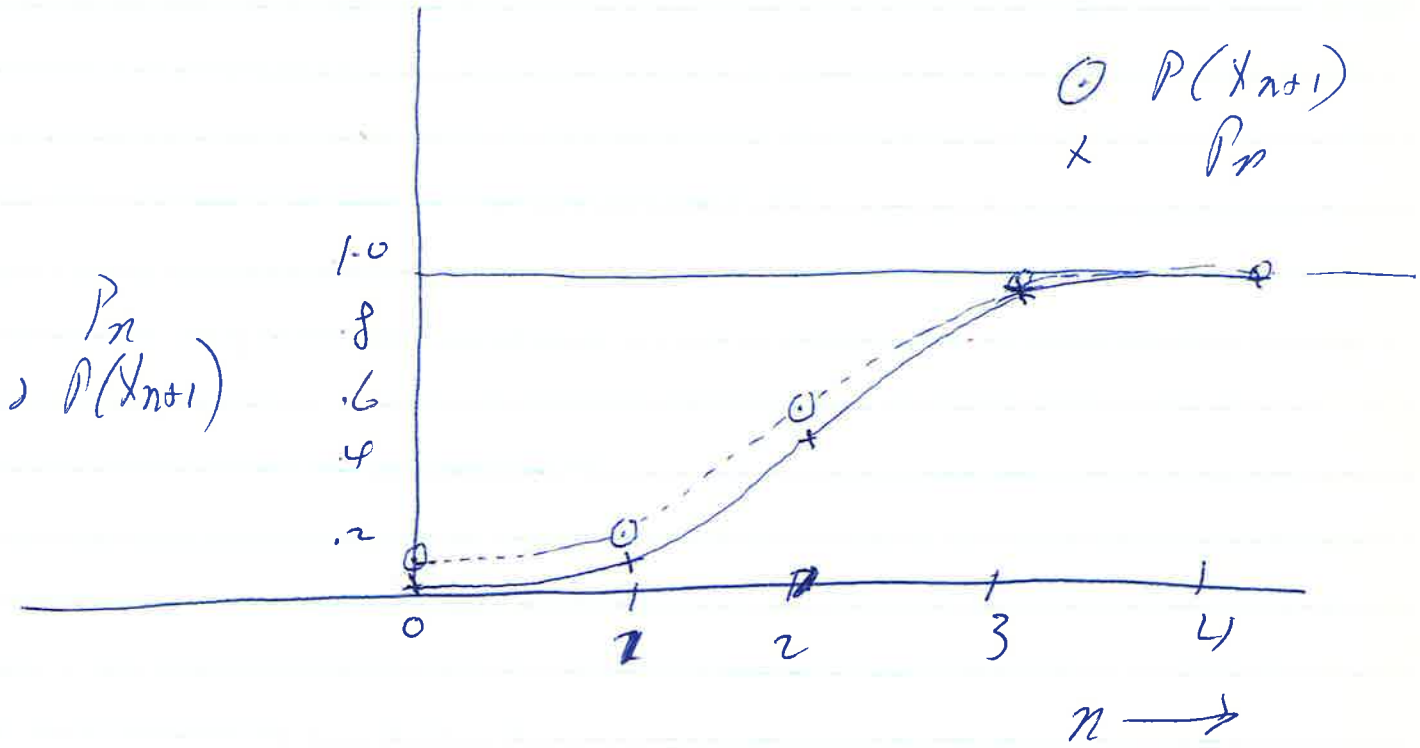
④ $P_n = \frac{1}{2}$ for $\varepsilon^n/x = 1$

⑤ $P_n = \varepsilon$ for $\varepsilon \approx \dfrac{1}{\varepsilon^n/x} = \dfrac{x}{\varepsilon^n}$

or $\dfrac{\varepsilon^{n+1}}{x} = 1$

We can sketch P_n and $P(X_{n+1})$ with $\varepsilon = .1$; $x = .01$





Ad hoc criterion If theory H is "related" w.r.t. X, H is ad hoc w.r.t. X

this means crit $\varepsilon = 1$, $\Rightarrow n^{(1)} = 1$ //

So there is no enhancement

non-ad hoc means small value ε

enhancement ratio behaves like $1/\varepsilon >> 1$

for $x \leq \varepsilon$

Notice also that if the theory makes a false prediction X then so a true theory cannot make a false prediction

we must have $P(\sim X/H) = 1$

and $P(X/H) = 0$

leading to $P(H/X) = 0$

So posterior probability of H is always zero if H is falsified by a prediction X, which is what we should expect.

But if $P(H/X) = 0$

Now write $H = C \,\&\, B_i$

then $P(H/X) = 0 \Rightarrow \begin{cases} P(B_i/C\,\&\,X) = 0 \\ \text{or } P(C/X) = 0 \text{ and } P(B_i/C\,\&\,X) \text{ is undefined} \end{cases}$

Suppose $P(B_i/C\,\&\,X)$ is undefined if C

So $P(C\,\&\,B_i) = P(B_i) \times P(C)$

If we know $P(C\,\&\,B_i) = 0$ but can we say about $P(B_i)$, $P(C)$.

then requires a methodological decision

Ad Lakatos, we take $P(B_i) = 0$, leave $P(C)$ unchanged

For new hypothesis B_2 the $P(C)$ as before and the calculation did $P(B_2) = \sqrt{\varepsilon}$,
where $P(B \& C / 2) = \sqrt{\varepsilon} \times \sqrt{\varepsilon} = \varepsilon$.

So $P(B_2 \& C) = \underbrace{\frac{P(B_2)}{\sqrt{\varepsilon}}}_{\text{"improved" value from previous enhancement using } B_1} \times P(C)$

with these value of $P(B_2 \& C)$ we can enhance with further predictions

Problem How do operation

$\Omega \times P(B \& C)$ between $P(B)$ & $P(C)$

when $P(B) \neq P(C)$ in order that constraints of probability calculus are not breached.

then we have $\Omega\, AB \angle 1$, $A \angle 1$, $B \angle 1$
in general

$= \Omega_A A \times \Omega_B B$

We want to choose Ω_A, Ω_B in that $\Omega_A A \angle 1$ and $\Omega_B B \angle 1$.

Notice now $P(C/X) = \sum_{i=1}^{N} P(C \& B_i / X)$, N possible B_i's.

If all $P(C \& B_i / X) = 0$ i.e. all $C \& B_i$ are falsified
then we know $P(C/X)$ must be zero so
methodological decision must always conform with conjunctive probability theory.